

SOLVED LOGIC PROBLEMS, Problem set F

Problems toward the end of this set are difficult.  
Extended rules are used on some problems to keep the  
solutions short.

1		$(x) Px$	P
2		$(x) \sim Qx / \sim (Ex) (Px \equiv Qx)$	
3		$(Ex) (Px \equiv Qx)$	A
4		a   $Pa \equiv Qa$	A
5		$(x) Px$	1, R
6		$Pa$	5, UE
7		$Qa$	4, 6, $\equiv E$
8		$(x) \sim Qx$	2, R
9		$\sim Qa$	8, UE
10		$Qa \vee \sim (Ex) (Px \equiv Qx)$	7, $\vee I$
11		$\sim (Ex) (Px \equiv Qx)$	9, 10, $\vee E$
12		$\sim (Ex) (Px \equiv Qx)$	3, 4-11, EE
13		$\sim (Ex) (Px \equiv Qx)$	3, 12, (3-12), $\sim I$

SET F PROB A

1		$(x) (Fx > Gx)$	P
2		$(x) (Gx > Hx)$	P
3		$\sim (Ex) Hx / \sim (Ex) Fx$	
4		$(Ex) Fx$	A
5		a   $Fa$	A
6		$(x) (Fx > Gx)$	1, R
7		$Fa > Ga$	6, UE
8		$Ga$	5, 7, $>E$
9		$(x) (Gx > Hx)$	2, R
10		$Ga > Ha$	9, UE
11		$Ha$	8, 10, $>E$
12		$(Ex) Hx$	11, EI
13		$(Ex) Hx$	4, 5-12, EE
14		$\sim (Ex) Hx$	3, R
15		$\sim (Ex) Fx$	13, 14, (4-14), $\sim I$

SET F PROB B

1		$\sim (x) (y) Lxy / (Ex) (Ey) \sim Lxy$	
2		$\sim (Ex) (Ey) \sim Lxy$	A
3		$(x) \sim (Ey) \sim Lxy$	2, $\sim X$

4		$\sim (Ey) \sim Lay$	3, UE
5		$(y) \sim \sim Lay$	4, $\sim X$
6		$\sim \sim Lab$	5, UE
7		$Lab$	6, $\sim E$
8		$(y) Lay$	7, UI
9		$(x) (y) Lxy$	8, UI
10		$\sim (x) (y) Lxy$	1, R
11		$\sim \sim (Ex) (Ey) \sim Lxy$	9, 10, (2-10), $\sim I$
12		$(Ex) (Ey) \sim Lxy$	11, $\sim E$

SET F PROB C

1		$\sim (Ex) (Ey) Lxy / (x) (y) \sim Lxy$	
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2		$(x) \sim (Ey) Lxy$	1, $\sim X$
3		$\sim (Ey) Lay$	2, UE
4		$(y) \sim Lay$	3, $\sim X$
5		$\sim Lab$	4, UE
6		$(y) \sim Lay$	5, UI
7		$(x) (y) \sim Lxy$	6, UI

SET F PROB D

1		$\sim (Ex) (Px \vee Qx) / (x) \sim Px \ \& \ (x) \sim Qx$	
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2		$(x) \sim (Px \vee Qx)$	1, $\sim X$
3		$\sim (Pa \vee Qa)$	2, UE
4		$\sim Pa \ \& \ \sim Qa$	3, DM
5		$\sim Pa$	4, $\&E$
6		$\sim Qa$	4, $\&E$
7		$(x) \sim Qx$	6, UI
8		$(x) \sim Px$	5, UI
9		$(x) \sim Px \ \& \ (x) \sim Qx$	7, 8, $\&I$

SET F PROB E

1		$\sim (x) (Px \ \& \ Qx) / (Ex) \sim Px \vee (Ex) \sim Qx$	
<hr/>			
2		$(Ex) \sim (Px \ \& \ Qx)$	1, $\sim U$
3	a	$\sim (Pa \ \& \ Qa)$	A
<hr/>			
4		$\sim Pa \vee \sim Qa$	3, DM
5		$\sim Pa$	A
<hr/>			
6		$(Ex) \sim Px$	5, EI
7		$(Ex) \sim Px \vee (Ex) \sim Qx$	6, $\vee I$
8			
9		$\sim Qa$	A
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10		$(Ex) \sim Qx$	9, EI
11		$(Ex) \sim Px \vee (Ex) \sim Qx$	10, $\vee I$
12			
13		$(Ex) \sim Px \vee (Ex) \sim Qx$	4, 5-7, 9-11, AC

14 |  $(\exists x) \sim Px \vee (\exists x) \sim Qx$  2, 3-13, EE

SET F PROB F

1 |  $(x) (\sim (Ey) Rxy \& \sim (Ey) Ryx) / (x) (y) \sim Rxy$

2 |  $\sim (Ey) Ray \& \sim (Ey) Rya$  1, UE

3 |  $\sim (Ey) Ray$  2, &E

4 |  $(y) \sim Ray$  3,  $\sim X$

5 |  $\sim Rab$  4, UE

6 |  $(y) \sim Ray$  5, UI

7 |  $(x) (y) \sim Rxy$  6, UI

SET F PROB G

1 |  $(\exists x) (Px \supset (y) (Py \supset Qy))$  P

2 |  $\sim (\exists x) Qx / \sim (x) Px$

3 | a |  $Pa \supset (y) (Py \supset Qy)$  A

4 | |  $(x) Px$  A

5 | |  $Pa$  4, UE

6 | |  $Pa \supset (y) (Py \supset Qy)$  3, R

7 | |  $(y) (Py \supset Qy)$  5, 6,  $\supset E$

8 | |  $Pa \supset Qa$  7, UE

9 | |  $Qa$  8, 5,  $\supset E$

10 | |  $(\exists x) Qx$  9, EI

11 | |  $\sim (\exists x) Qx$  2, R

12 | |  $\sim (x) Px$  10, 11, (4-11),  $\sim I$

13 |  $\sim (x) Px$  1, 3-12, EE

SET F PROB H

1 |  $(Ey) (Ez) ((x) \sim Rxy \vee (x) \sim Rxz) / \sim (y) (z) (\exists x) (Rxy \& Rxz)$

2 | |  $(Ez) ((x) \sim Rxav (x) \sim Rxz)$  A

3 | | |  $(x) \sim Rxa \vee (x) \sim Rxb$  A

4 | | |  $(y) (z) (\exists x) (Rxy \& Rxz)$  A

5 | | | |  $(z) (\exists x) (Rxa \& Rxz)$  4, UE

6 | | | |  $(\exists x) (Rxa \& Rxb)$  5, UE

7 | | | |  $Rca \& Rcb$  A

8 | | | |  $Rca$  7, &E

9 | | | |  $Rcb$  7, &E

10 | | | |  $(x) \sim Rxa \vee (x) \sim Rxb$  3, R

11 | | | |  $(x) \sim Rxa$  A

12 | | | |  $\sim Rca$  11, UE

13 | | | |  $Rca$  8, R

14 | | | |  $Rcav \sim (y) (z) (\exists x) (Rxy \& Rxz)$  13, vI

15						$\sim (y) (z) (Ex) (Rxy \& Rxz)$	12, 14, vE
16							
17						$(x) \sim Rxb$	A
18						$Rcb$	9, R
19						$\sim Rcb$	17, UE
20						$Rcb \vee \sim (y) (z) (Ex) (Rxy \& Rxz)$	18, vI
21						$\sim (y) (z) (Ex) (Rxy \& Rxz)$	19, 20, vE
22							
23						$(x) \sim Rxa \vee (x) \sim Rxb$	3, R
24						$\sim (y) (z) (Ex) (Rxy \& Rxz)$	23, 11-15, 17-21, AC
25						$\sim (y) (z) (Ex) (Rxy \& Rxz)$	6, 7-24, EE
26						$\sim (y) (z) (Ex) (Rxy \& Rxz)$	4, 25, (4-25), ~I
27						$\sim (y) (z) (Ex) (Rxy \& Rxz)$	2, 3-26, EE
28						$\sim (y) (z) (Ex) (Rxy \& Rxz)$	1, 2-27, EE

SET F PROB I

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